# The Principle of Base

Different cultures used different methods to count and represent numbers. One of the most universal ways was to use the fingers on your hand. Other cultures used the toes on their feet and other body parts to count and represent numbers. Beads and pebbles were also often used to count and keep track of things.  As cultures became more literate, they began to assign symbols to the numbers so that they could write the number down. There are two different procedures that people can use to symbolize the concept of number. The two procedures are the cardinal and ordinal procedures.  The cardinal procedure involves using one symbol which represents the number one and using that symbol as often as needed. The ordinal procedure assigns a different symbol for each whole number. The problem with the cardinal approach is that it is not very efficient. It is difficult to see at a glance what the number that is written down is supposed to symbolize. The problem with the ordinal approach is that an infinite number of numerical symbols have to be created.  The solution falls somewhere in between these two approaches where all numbers can be represented with relatively few symbols. (p. 31)

This was done by creating a base number system. The most prevalent base number system today is the decimal system or base ten. It has been almost universally adopted.  This is no doubt to the fact that people first learned how to count using their ten fingers.  This makes base 10 anatomically convenient but the decimal system has “few advantages from a mathematical or practical viewpoint.” (p. 34).  If the choice for a base had been left up to the practical man, he would have insisted on a base with the greatest number of divisors such as twelve. The number twelve has four divisors while ten only has two. The mathematician would argue for a prime number such as seven or eleven. The great mathematician Lagrange stated that a prime base meant that “every systematic fraction would be irreducible and would therefore represent the number in a unique way” (p. 35). All the number bases suggested have the advantage of falling in a “range that does not strain human memory” which number bases such as 20 or 60 would.  These number bases are also more convenient to write out than small number bases such as 2 or 3 (p. 35-36).

The decimal has been almost universally adopted today. In the past there were a number of cultures that used a different base number system. Instead of using only their fingers to count, some cultures used their fingers and their toes. This meant that they used a base 20 number system which is commonly known as the vigesimal system. The Mayan, Aztec and Celtic cultures all used the vigesimal system for counting. It is from the Celtic language that we get the word score being used for the number twenty.  The Bible has several examples of score being used to count.  Other cultures use base quinary system or a base 5 number system. There are merchants today in the Bombay region of India that use the quinary system. They count using their left hand to represent single units while the fingers on their right hand mark each group of five units (p. 36). The quinary and vigesimal systems have links to the human anatomy but another historical base number system does not.  This is the sexagesimal system or the base 60 number system. This was used by the Sumerians and Babylonians. It is still used today for measuring time arcs and angles. One of the reasons for the use of sexagesimal system argued that 60 is the lowest number with a great many divisors (p. 51).

The invention of computers saw the binary system or the base 2 number system become predominant. This was due to the electrical nature of computers where a switch can be either on or off. The binary system has only two numbers with the 0 representing off and the 1 representing on. The problem with binary number is that they can become fairly lengthy so computer programmers have used octal (base 8) and hexadecimal (base 16) to reduce the number of digits needed to represent a binary number.

The question remains what exactly is meant with a base number system. This can easily be explained by looking at decimal system which everybody is familiar with. With the decimal system there are 10 unique digits (0,1,2,3,4,5,6,7,8,9). In the binary system there are 2 unique digits (0,1). With these unique digits, every number large and small can be represented. This is done by starting on the right of a whole number and moving left. The digit furthest to the right represents the ones or $10^0$. Moving left there is the tens ($10^1$) and then hundreds ($10^2$) and so on. Each time we move to the left the base number is raised one more power. The digit in that position is multiplied by power of the base. For example the decimal number 234 is evaluated as

$$(2 \times 10^2) + (3 \times 10^1) + (4 \times 10^0)$$
$$= 200 + 30 + 4$$
$$= 234$$

We could represent the same number in quinary (base 5) using the same technique.

$$(1 \times 5^3) + (4 \times 5^2) + (1 \times 5^1) + (4 \times 5^0)$$
$$= 125 + 100 + 5 + 4$$
$$= 234$$

Thus the number 234 in decimal number system is written 1414 in quinary number system. This can be done in any number system chosen. The *Number Base Convertor* allows you to explore this.

**A List of the name of different number systems**

1 - **unary**
2 - **binary**
3 - **ternary** / trinary
4 - **quaternary**
5 - **quinary** / quinternary
6 - **senary** / **heximal** / hexary
7 - **septenary** / septuary
8 - **octal** / octonary / octonal / octimal
9 - **nonary** / novary / noval
10 - **decimal** / **denary**
11 - **undecimal** / undenary / unodecimal
12 - **dozenal** / duodecimal / duodenary

13 - **tridecimal** / **tredecimal** / triodecimal
14 - **tetradecimal** / quadrodecimal / quattuordecimal
15 - **pentadecimal** / quindecimal
16 - **hexadecimal** / sexadecimal / sedecimal
17 - septendecimal / heptadecimal
18 - octodecimal / decennoctal
19 - nonadecimal / novodecimal / decennoval
20 - **vigesimal** / bigesimal / bidecimal
21 - unovigesimal / unobigesimal
22 - duovigesimal
23 - triovigesimal
24 - **quadrovigesimal** / quadriovigesimal
26 - **hexavigesimal** / sexavigesimal
27 - heptovigesimal
28 - octovigesimal
29 - novovigesimal
30 - **trigesimal** / triogesimal

Ifrah, Georges. *From One to Zero: A Universal History of Numbers.* New York: Viking Penguin Inc., 1985